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# Nonlinear Intermodulation of two Coherent Acoustic Progressive Waves Emitted by a Large Bandwidth Loudspeaker.

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## Abstract

The main purpose of the paper is to contribute at presenting a formalism which would be relevant for interpreting qualitatively the nonlinear harmonic distortion, and more particularly the strong intermodulation, which occur in air in the frequency range (20 Hz-20 kHz) when the wave is propagating.

For that purpose, the one-dimensional nonlinear acoustic propagation in infinite medium of two harmonic waves, at frequencies respectively equal to several Hz and several kHz, is dealt with. Despite the experimental uncertainties which result from the nature of the field emitted by the large bandwidth loudspeaker used, it can be concluded especially that intermodulation due to nonlinear effects when the acoustic wave propagates provides important non harmonic distortion in usual listening conditions.

## I. INTRODUCTION

**I**t is well known that the high fidelity sound reproduction is very difficult to achieve because it involves many nonlinear effects [1]. Regarding the nonlinearities of the loudspeakers themselves, papers are concerned with the causes of these nonlinearities [2] and their nature (eddy currents [3], suspension and non homogeneous magnetic field [4], and others). The purpose of a recently published paper is to contribute at diminishing some of these nonlinearities, particularly those arising from the suspension and

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the motor [5],[6]. A small loudspeaker (5 *cm* in diameter) has been designed which allows to investigate the effects of other nonlinear phenomena, namely those occurring when the wave propagates. Several effects have been invoked to explain both observed harmonic distortion and intermodulation distortion generated by loudspeakers [7],[8],[9],[10],[11],[12],[13]. But the contribution of nonlinear effects due to both the nonlinear properties of the medium and the nonlinear behaviour occurring at high acoustic pressure level was not considered so far to interpret the intermodulation phenomena.

The literature abounds with many papers in which topics involving nonlinear propagation of harmonic signals are of principal focus. These topics have been discussed a long time ago (see for example [14],[15],[16]) and are also currently discussed in standard textbooks [17],[18],[19], which provide methodology for predicting acoustic properties when nonlinear effects are taken into account. Analytical works deal primarily with methods which account for both nonlinear effects due to the finite amplitude of the wave (the material derivative with respect to the spatial coordinates does not reduce to the partial derivative) and the nonlinear behaviour of the fluid when it experiences pressure variations (pressure variations are no more proportional to density variations). They also account for the dissipative processes due to heat conduction and viscosity, among others.

Regarding the analytical, numerical and experimental methods for describing the one-dimensional acoustic propagation in an unbounded medium, papers are usually concerned with the shape profile perturbation of the wave when propagating, with the generation of harmonic components of the signal created by the source, and also with combinations of two harmonic signals created by a source to generate a wave at a much lower frequency than both frequencies of the primary signals (i.e. the difference of the frequencies created by the source).

But an analytic and experimental procedure whereby two harmonic acoustic waves, one at very low frequency and the other one at a much higher frequency, can generate several observable frequencies which are linear combinations of the primary frequencies, when nonlinear acoustic phenomena in fluid media are taken into account, providing an understanding that strong intermodulations can occur in gases and then providing methodology for predicting acoustical properties of complex but coherent sound fields emitted by a sound source, leads to situations not considered so far in the framework of the design of current, wide frequency range loudspeakers.

Therefore, in the present paper, the one-dimensional acoustic propagation in infinite medium of two harmonic waves, at frequencies respectively equal to several Hz and several kHz, emitted by a wide band, quasi linear medium loudspeaker, using an analytical method relying on the well-known nonlinear Fubini's theory [12],[13] (inviscid and non heat-conducting fluid) and in some respect (extending in range for both frequencies) Fenelon's works [20],[21], is dealt with. Therefore the main goal of the paper is to contribute at emphasising that the intermodulation phenomena when the wave propagates reduce in practice the frequency range of a single loudspeaker, this limitation being particularly strong for loudspeakers which allow a great displacement amplitude and thus a high sound level with a relatively small membrane. It is worth noting that the theoretical part of the present paper assumes a plane wave propagation because, in the experiment, the shape of each wave depends strongly on the frequency (accurate interpretation of these complex waves would provide more complex analytical solutions which would overshadow the purpose of the paper, even they would assume drastic approximations).

## II. NON LINEAR FUNDAMENTAL EQUATIONS

The one dimensional problem addressed here governs the behaviour of an acoustic field in infinite domain, assumed to be excited by two time-periodic sources at the same location, here at  $x = 0$  (in fact one loudspeaker providing two signals), in the framework of the nonlinear acoustics, the fluid being inviscid and non heat-conducting. The parameters which specify the properties and the nature of the fluid are the ambient values of the density  $\rho_0$ , the mean pressure  $P_0$ , the adiabatic speed of sound  $c_0$ , and the specific heat ratio  $\gamma$ . The variables describing the dynamic and the thermodynamic state of the fluid are the pressure variation  $p = P - P_0$ , the particle velocity (along the x-axis)  $v$ , and the density variation  $\rho' = \rho - \rho_0$ . A complete set of linearised, homogeneous (outside the source) equations governing disturbances of the fluid includes the following (equations 10.54 to 10.60 in [17]):

- the Euler equation

$$\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial x} + \rho \frac{\partial v}{\partial x} = 0 \quad , \quad (1)$$

- the conservation of mass equation

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \rho^{-1} \frac{\partial p}{\partial x} = 0 \quad , \quad (2)$$

- the adiabatic behaviour of the gas

$$\frac{\partial p}{\partial x} = c^2 \frac{\partial \rho}{\partial x} , \quad (3)$$

where the parameter  $c$ , defined as

$$c = c_0 \left( 1 \pm \frac{\gamma - 1}{2} \frac{v}{c_0} \right) \quad (4)$$

depends on the unknown particle velocity field  $v$ , the parameter  $\gamma - 1 = \frac{\chi_T - \chi_S}{\chi_S}$  giving a relative "measure" of the "gap" between the isothermal compressibility  $\chi_T$  and the adiabatic compressibility  $\chi_S$ .

Combining equations 1 to 3 with equation 4 to remove the variables  $p$  and  $\rho$  leads to the nonlinear propagation equations governing the counter-propagating particle velocity waves:

$$\frac{\partial v}{\partial t} \pm c_0 \left( 1 \pm \frac{\gamma + 1}{2} \frac{v}{c_0} \right) \frac{\partial v}{\partial x} = 0 . \quad (5)$$

This result conveys the interpretation of the function

$$c_w = c_0 \left( 1 \pm \frac{\gamma + 1}{2} \frac{v}{c_0} \right) \quad (6)$$

as the speed of sound, which depends on the unknown particle velocity field  $v$  through the nonlinear parameter  $\frac{\gamma + 1}{2}$ .

#### A. Single frequency source activity

The solution of these equations, for harmonic plane motion ( angular frequency  $\omega$ ), takes the well-known form (Fubini's solution) given in Appendix A. As the wave propagates, distortion of its profile with respect to the distances from the loudspeaker appears, until a shock wave (amplitude discontinuity) occurs at a critical distance,  $\widetilde{x}_\omega$  (equation 10 in Appendix A)

This critical distance  $\widetilde{x}_\omega$  is presented in figure 1 as a function of the frequency, for two particle velocities given at the source location  $x = 0$ , namely  $1m/s$  (full line) and  $0.1m/s$  (dashed line), showing that this critical distance decreases when both the frequency and the initial particle velocity increase. As a result, when the frequency is equal to  $5kHz$  and the initial particle velocity amplitude is equal to  $0.1m/s$  (that is for an acoustic pressure level equal to 126 dB SPL), the shock wave appears at  $\widetilde{x}_\omega \simeq 30m$  which is quite far from the loudspeaker compared to the usual listening distance, but the shape of the wave

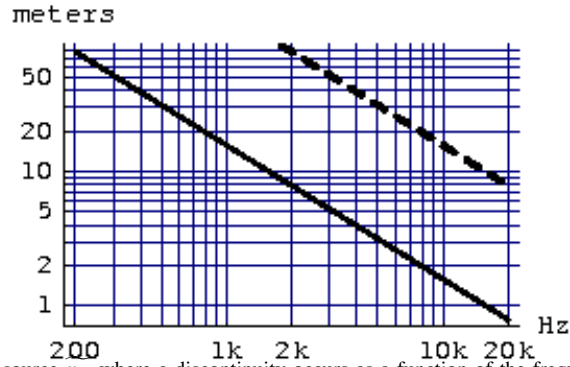


Fig. 1. Distance from the source  $x_\omega$  where a discontinuity occurs as a function of the frequency, for two particle velocities given at the source location  $x = 0$ , namely  $1\text{ m/s}$  (full line) and  $0.1\text{ m/s}$  (dashed line).

becomes significantly modified only after a few meters propagating. Therefore it can be emphasized that "high fidelity" sound reproduction is physically impossible, even if transducers would be perfect, when high particle velocity ( or pressure level) is required, except very near the source (which is not really recommended for such levels!).

#### B. Double frequency source activity

The solution of the fundamental equations when two harmonic waves are created by the source is given in Appendix B (equations 25 to 28). It differs here from those available in the literature [20][21] because the present paper considers that the frequencies are independent. The results show not only the generation of harmonic components of each wave separately, as expected, but also the nonlinear intermodulation between these two harmonic waves. The following section shows that under some circumstances a strong intermodulation phenomenon can occur.

### III. THEORETICAL RESULTS

Being concerned by the nonlinear wave distortion when the wave propagates away from the source, we give in this section the results for the nonlinear distortion of a harmonic wave and the nonlinear intermodulation between two harmonic waves created by the loudspeaker. These theoretical results mainly concern the amplitude of generated signals at frequencies which are combinations of the primary frequencies.

An example of the contribution of the nonlinear effects to the generation of harmonic components of the signal created by the source, calculated from Fubini's solution (11), is given in figure 2, showing the amplitude of the  $n^{th}$  harmonics ( $n = 0$  to 5) in dB rear the amplitude of the fundamental ( $n = 0$ ), as a function of the integer  $n$ , for three values of the normalised distance  $\sigma_\omega$  from the source, namely the distance of the creation of the shock wave  $\sigma_\omega = 1$  (upper dots),  $\sigma_\omega = 0.5$  (middle dots) and  $\sigma_\omega = 0.1$  (lower dots). As an example, the level of the first harmonic at a distance half of the critical distance ( $\sigma_\omega = 0.5$ ) is only 27 dB lower than the level of the fundamental signal created by the loudspeaker, which is non negligible in the usual listening conditions.

Another example of harmonic generation is given in figure 3 and table I, for a harmonic source vibrating with a velocity amplitude equal to 1 m/s, providing an acoustic level of 146 dB SPL. At a distance of one meter from the source, for a fundamental frequency of 110 Hz, the second harmonic (220 Hz) reaches 97 dB SPL, which is almost 50 dB SPL under the fundamental level. At 5 kHz, the second harmonic (10 kHz) reaches 130 dB SPL, which is only 16 dB SPL under the fundamental level.

The second set of theoretical results presented here involves the contribution of the nonlinear effects to the nonlinear intermodulation between two harmonic waves created by the loudspeaker, at a distance

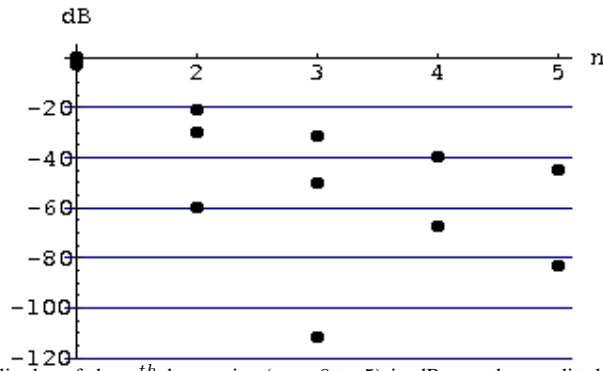


Fig. 2. Theoretical amplitudes of the  $n^{th}$  harmonics ( $n = 0$  to 5) in dB rear the amplitude of the fundamental ( $n = 0$ ), as a function of the integer  $n$ , for three values of the normalised distance  $\sigma_\omega$  from the source, namely the distance of the creation of the shock wave  $\sigma_\omega = 1$  (upper dots),  $\sigma_\omega = 0.5$  (middle dots) and  $\sigma_\omega = 0.1$  (lower dots).

from the loudspeaker equal to  $x = 1$  m (corresponding to the experimental results presented below in the next section). The particle velocity spectrum (dB) is presented in figures 3 and 4 as a function of the frequency, for two harmonic particle velocity waves generated simultaneously by the source located  $x = 0$ , at frequencies respectively equal to  $\omega/2\pi = 110$  Hz and  $\Omega/2\pi = 5$  kHz, at levels both equal to 1 m/s in figure 3 and respectively equal to 5 m/s and 1 m/s in figure 4. The contribution of the four terms of the solution (24), namely  $S_\Omega$  (equation 25),  $S_\omega$  (equation 26),  $D_{\Omega,n\omega}$  (equation 27),  $D_{\omega,n\Omega}$  (equation 28), can be clearly identified on each figure. Very high speeds are considered in order to emphasise the intermodulation phenomena.

Looking at figure 4 for example, the points at the abscissa 5 kHz, 10 kHz, 15 kHz, and 20 kHz correspond to a harmonic generation (term  $S_\Omega$ , equation 25), 5 kHz being the fundamental frequency generated by the source. The points at the abscissa 110 Hz, 220 Hz, 330 Hz, 440 Hz, and 550 Hz correspond to the other harmonic generation (term  $S_\omega$ , equation 26), 110 Hz being the fundamental frequency generated by the source. The points at the abscissa  $(5000 \pm 110)$  Hz,  $(5000 \pm 220)$  Hz,  $(5000 \pm 330)$  Hz, and  $(5000 \pm 440)$  Hz correspond to a nonlinear intermodulation (term  $D_{\Omega,n\omega}$ , equation 27), 5 000 Hz and 110 Hz being the fundamental frequencies generated by the source. And finally, the points at the abscissa  $(110 + 5000)$  Hz,  $(110 + 10000)$  Hz, and  $(110 + 15000)$  Hz correspond to the other nonlinear intermodulation (term  $D_{\omega,n\Omega}$ , equation 28), 110 Hz and 5000 Hz being the fundamental frequencies generated by the source.

The main feature is that the first order intermodulation signal, at  $(5000 \pm 110)$  Hz, is very important, regarding the small distance (1 m) from the loudspeaker considered, because its level is only 16 dB lower than the level of the fundamental signals (at 110 Hz and 5 kHz respectively) for the case depicted on figure 3, and because its level is only 6 dB lower than the level of the fundamental signals (the one at 5 kHz) for the case depicted on figure 4. These results are mainly governed by the amplitude of the term  $D_{\Omega,n\omega}$  which is proportional to the fundamental frequency  $\Omega/2\pi = 5$  kHz, and also to the amplitude of the original signals and the distance from the source.

Then, even when the distance from the loudspeaker considered is very small (1 m, i.e. far from the



distance of the generation of the shock wave), it is remarkable that the nonlinear intermodulation cannot be negligible. Actually, because it is proportional to both amplitudes of the original signals (at the angular frequencies  $\Omega$  and  $\omega$ ), this nonlinear effect becomes negligible when these levels are much lower than those considered here.

More details are given in table 1, giving the particle velocities (dB) as functions of the frequencies, for two harmonic particle velocity waves generated simultaneously by a source one meter far from the observation point, at frequencies respectively equal to  $\omega/2\pi = 110$  Hz and  $\Omega/2\pi = 5$  kHz, at levels both equal to 1 m/s in the case one (corresponding to figure 3) and respectively equal to 5 m/s and 1 m/s in the case 2 (corresponding to figure 4).

|                      | Frequencies  | case 1: Fig.3 | case 2: Fig.4 |
|----------------------|--------------|---------------|---------------|
| $S_{\Omega}$         | 5 kHz        | 0 dB (1m/s)   | 0 dB (1m/s)   |
| $S_{\omega}$         | 110 Hz       | 0 dB (1m/s)   | +14 dB (5m/s) |
| $S_{2\omega}$        | 220 Hz       | -49           | -20           |
| $S_{3\omega}$        | 330 Hz       | -95           | -52           |
| $S_{2\Omega}$        | 10 kHz       | -16           | -16           |
| $S_{3\Omega}$        | 15 kHz       | -29           | -29           |
| $D_{\Omega,1\omega}$ | 4890-5110 Hz | -16           | -2            |
| $D_{\Omega,2\omega}$ | 4780-5220 Hz | -65           | -37           |
| $D_{\Omega,3\omega}$ | 4670-5330 Hz | -110          | -68           |
| $D_{\omega,1\Omega}$ | 5110 Hz      | -49           | -35           |
| $D_{\omega,2\Omega}$ | 10110 Hz     | -65           | -51           |

TABLE I

THE PARTICLE VELOCITIES (dB) AS FUNCTIONS OF THE FREQUENCIES, FOR TWO HARMONIC PARTICLE VELOCITY WAVES GENERATED SIMULTANEOUSLY BY A SOURCE 1 METER FAR FROM THE OBSERVATION POINT, AT FREQUENCIES RESPECTIVELY EQUAL TO  $\omega/2\pi = 110$  Hz AND  $\Omega/2\pi = 5$  kHz, AT LEVELS BOTH EQUAL TO 1 M/S IN THE CASE 1 (CORRESPONDING TO FIGURE 3) AND RESPECTIVELY EQUAL TO 5 M/S AND 1 M/S IN THE CASE 2 (CORRESPONDING TO FIGURE 4). THE LEVEL 1M/S IS CONSIDERED AS THE REFERENCE ONE.

In order to show the contribution of a distortion of the signal created by the source (due to source nonlinearities) to the nonlinear intermodulation due to the nonlinear propagation effects, three signals are assumed to be generated simultaneously by the source, two of them at the same frequencies as those considered before (110 Hz and 5 kHz) and the third one at the first harmonic of the lower frequency,

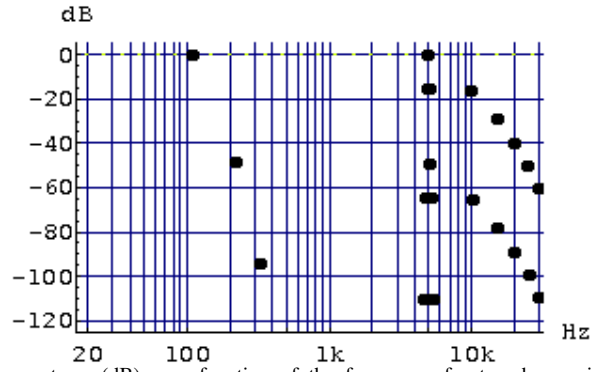


Fig. 3. Particle velocity spectrum (dB) as a function of the frequency, for two harmonic particle velocity waves generated simultaneously by a source 1 meter far from the observation point, at frequencies respectively equal to  $\omega/2\pi = 110$  Hz and  $\Omega/2\pi = 5$  kHz, at levels both equal to 1 m/s. This level is considered as the reference one.

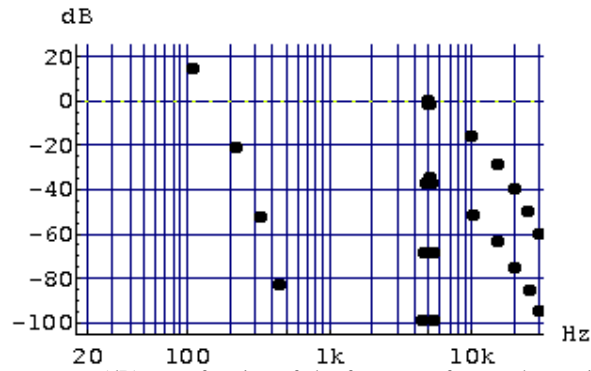


Fig. 4. Particle velocity spectrum (dB) as a function of the frequency, for two harmonic particle velocity waves generated simultaneously by a source one meter far from the observation point, at frequencies respectively equal to  $\omega/2\pi = 110$  Hz and  $\Omega/2\pi = 5$  kHz, at levels respectively equal to 5 m/s and 1 m/s. The latter level is considered as the reference one.

namely 220 Hz, the level of this last signal being 26 dB lower than the signal at 110 Hz.

Figure 5 shows the particle velocity spectrum (dB) as a function of the frequency when the source generates a distorted low frequency signal, with the fundamental (110 Hz, not shown) and the first harmonic (220 Hz, 0.05 m/s), and the high frequency signal (5 kHz, 1 m/s).

The contribution of the four terms of the solution (24), namely  $S_\Omega$  (equation 25),  $S_\omega$  (equation 26),  $D_{\Omega,n\omega}$  (equation 27),  $D_{\omega,n\Omega}$  (equation 28), can be clearly identified on figure 3 as on figures 4 and 5. The intermodulation signals, at  $(5000 \pm 110)$  Hz, are non negligible, regarding the small distance (1 m) from the loudspeaker considered.

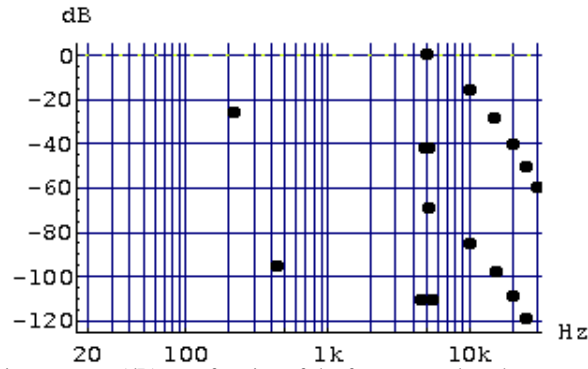


Fig. 5. The particle velocity spectrum (dB) as a function of the frequency, when the source creates simultaneously three signals, respectively at 110 Hz (not shown), at 220 Hz (the first harmonic due to the distortion, 0.05 m/s), and at 5 kHz (1 m/s). The level 1m/s is considered as the reference.

#### IV. EXPERIMENTAL RESULTS AND CONCLUSION

Various experimental results given below serve to confirm the estimate of the nonlinear intermodulation, especially the first order intermodulation, predicted by the modelling lying on the Fubini's theory presented in the previous sections. Electrical energy, containing two harmonic signals, one at 110 Hz and the other one at 5 kHz, is provided to an electrodynamic loudspeaker. But the velocity field of the membrane of the loudspeaker (50 mm in diameter) incorporates nonlinear features because both signals are generated at a quite high level. Results of measurements of the velocity of the membrane, using a laser vibrometer, are shown in figure 6: they reveal that the spectrum contains several harmonics of the fundamental electric signals, the more important ones being associated to the lowest frequency because the associated displacement is much greater than the displacement associated to the highest frequency (the first peak at 50 Hz is due to the electric amplifier). Actually, this harmonic distortion and the complex shape of the wave emitted by the acoustic source will prevent us to derive results with sufficiently accurate precision to obtain fine quantitative conclusions; nevertheless the measurements furnish results with orders of magnitude which permit to support qualitatively the theoretical results, as mentioned below.

For purposes of justifying this proposition, figures 7, 8, and 9 depict the spectral components of the acoustic pressure measured at the observation point one meter far from the loudspeaker on its axis.

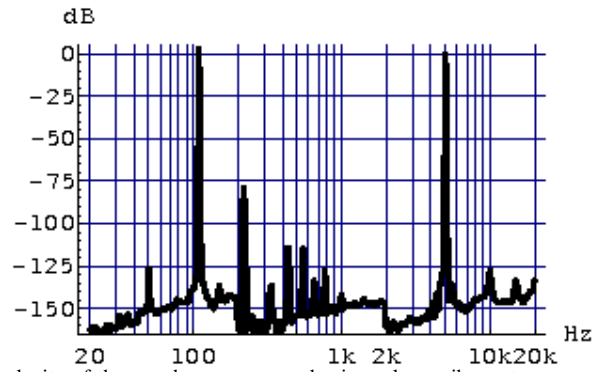


Fig. 6. Spectrum of the velocity of the membrane, measured using a laser vibrometer.

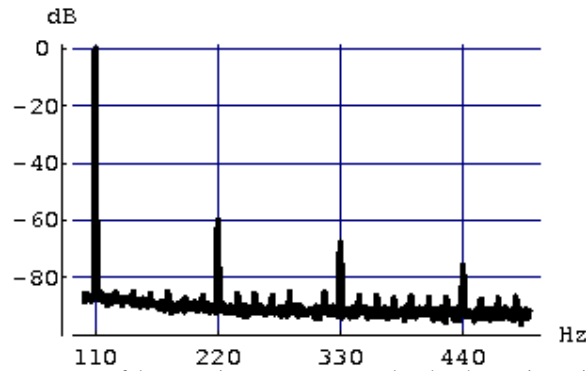


Fig. 7. Spectral harmonic components of the acoustic pressure measured at the observation point one meter far from the loudspeaker on its axis, on the frequency range (100 Hz, 500 Hz).

The harmonic components (220 Hz, 330 Hz, and so on) of the low frequency initial signal (110 Hz) which appear on figure 7 characterise both the nonlinearity of the loudspeaker emission and the nonlinearity occurring as the wave propagates (term  $S_\omega$  from equation 26), one of them not being able to be departed from the other one here.

The nonlinear intermodulation effects are shown in figure 8. First, as expected, the first order intermodulation signal, at  $(5000 \pm 110)$  Hz, which corresponds to the term  $D_{\Omega,1\omega}$  in the modelling, is the most important one, its level being roughly 30 dB lower than the level of the fundamental signals (5 kHz, 125 dBSPL); this result is totally coherent with the corresponding theoretical result (see the theoretical intermodulation signal on figure 3 and table 1) because the experimental level of the low frequency (110

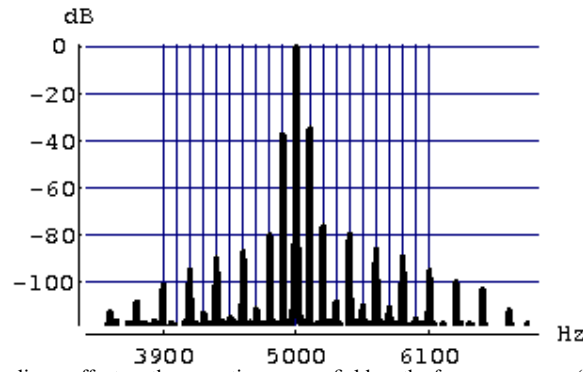


Fig. 8. Inter-modulation nonlinear effect on the acoustic pressure field on the frequency range (3 kHz, 7 kHz) around the fundamental frequency at 5 kHz.

Hz) signal is 25 dB lower than the level of the high frequency (5 kHz) when they have the same level in the results presented in figure 3.

Second, the components at  $(5000 \pm 220)$  Hz can correspond either to the second order term of the solution  $D_{\Omega, 2\omega}$  in the modelling associated to the fundamental frequencies 5kHz and 110 Hz, or to the first order term of the solution  $D_{\Omega, 1\omega}$  in the modelling associated to the fundamental frequency 5kHz and the first harmonic (220 Hz) of the fundamental (110 Hz) due to the nonlinear behaviour of the loudspeaker (in any case, both represent an intermodulation phenomenon arising from the nonlinear propagation effects). Finally, the upper order intermodulation effects, namely at  $(5\,000 \pm n \cdot 110)$  Hz with  $n \geq 3$ , exist but their levels decrease quite rapidly.

The first harmonic component (at 10 kHz) of the fundamental frequency (at 5 kHz), and the frequencies arising around it by the intermodulation phenomena, both being essentially due to the nonlinear propagation of the wave, are shown in figure 9, which gives their amplitudes as functions of the frequency.

To conclude, it can be said that this work has highlighted the importance of the nonlinear propagation phenomena, modelled in the framework of Fubini's theory, in having given results concerning not only the well known generation of harmonics but also important intermodulation effects. Actually, the harmonic distortion of the loudspeaker used in the experiments and the local effects at its surrounding (for example edge effects not evaluated here) will prevent us to derive results with sufficiently accurate precision to

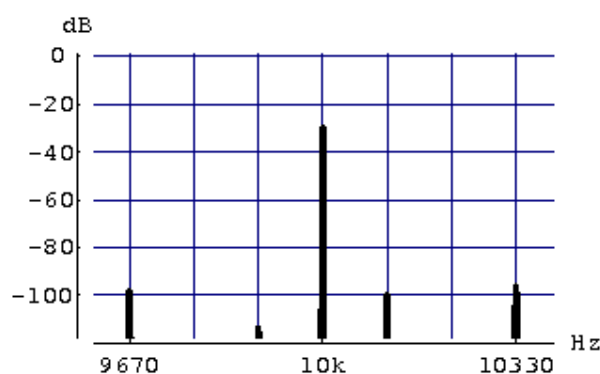


Fig. 9. Harmonic component (at 10 kHz) of the fundamental frequency (at 5 kHz), and the frequencies arising around it by the intermodulation phenomena, in the frequency range (9.5 kHz, 10.5 kHz).

obtain fine quantitative conclusions. Nevertheless, despite the discrepancies and the uncertainties which result from these features, it can be concluded that there is seen to be qualitative agreement between analytical and experimental results, thereby supporting the fact that not only the harmonic distortion is non negligible at high level, but also that intermodulation, due to nonlinear effects when the acoustic wave propagates, provides important non harmonic distortion in usual listening conditions.

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## APPENDIX

In the problem addressed in this appendix, the harmonic source set at  $x = 0$  creates a wave at the angular frequency  $\omega$  which propagates in the direction of the x-axis, the particle velocity at  $x = 0$  being given by:

$$v(0, t) = V_\omega \sin \omega t \quad . \quad (7)$$

The solution of equation 5, subject to the boundary condition 7, is given by (Fubini's solution)

$$v(x, t) = V_\omega \sin \omega \left\{ t - \frac{x}{c_0} \left( 1 + \frac{\gamma + 1}{2} \frac{v}{c_0} \right)^{-1} \right\} \quad . \quad (8)$$

Usually, this result can be truncated to include only the first order expansion with respect to the small non-dimensional parameter  $\left( \frac{\gamma + 1}{2} \frac{v}{c_0} \right)$ , leading to

$$v(x, t) \simeq V_\omega \sin \omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma + 1}{2} \frac{v}{c_0} \right) \right\} \quad . \quad (9)$$

As the wave propagates, distortion of its profile with respect to time appears, until a discontinuity occurs (generation of higher harmonics and shock wave) at the distance from the source  $\widetilde{x}_\omega$  which is given by equation (10) expressing that the difference between the distance covered by a neutral point of the wave ( $v = 0$ ) and the distance covered by the crest of the wave (maximum of the amplitude of the wave) at the same time is equal to a quarter wavelength, namely:

$$\widetilde{x}_\omega \simeq \frac{2c_0^2}{(\gamma + 1)\omega V_\omega} \quad . \quad (10)$$

It should be noted that, beyond this critical point, solution (9) has no physical meaning because it is a multi-valued function with respect to  $x$  (when analysing the phenomena in the neighbouring or beyond this point, the dissipative processes must be taken into account because they become predominant at the vicinity of the discontinuity).

This critical distance  $\widetilde{x}_\omega$  is presented in figure 1 as a function of the frequency, for two particle velocities given at the source location  $x = 0$ , namely  $1m/s$  (full line) and  $0.1m/s$  (dashed line), showing that this critical distance decreases when both the frequency and the initial particle velocity increases. As a result,



when the frequency is equal to  $5kHz$  and the initial particle velocity amplitude is equal to  $0.1m/s$  (that is for an acoustic pressure level roughly equal to 126 dB SPL), the shock wave appears at  $\widetilde{x}_\omega \simeq 30m$  which is quite far from the loudspeaker compared to the usual listening distance, but the shape of the wave becomes significantly modified only after several meters propagating (as shown in sections 3 and 4).

In the present paper, the distance from the source considered is always much lower than this critical distance  $\widetilde{x}_\omega$ , so that the dissipative effects can be neglected. Then, solution (9) can also take the form of a Fourier series as follows:

$$v(x, t) = V_\omega \sum_{n=1}^{\infty} \frac{2}{n\sigma_\omega} J_n(n\sigma_\omega) \sin \left\{ n\omega \left( t - \frac{x}{c_0} \right) \right\} \quad (11)$$

where the functions  $J_n$  are the Bessel functions of the first kind, with

$$\sigma_\omega = \frac{x}{\widetilde{x}_\omega} < 1 \quad . \quad (12)$$

This result shows clearly that the nonlinear phenomena lead to energy transfer from the signal created by the source (at the fundamental frequency) to harmonic components (generating the shape profile distortion of the wave when propagating).

In this appendix, the source set at  $x = 0$  is assumed to create two harmonic waves, at the angular frequencies denoted  $\omega$  and  $\Omega$ , their particle velocity amplitudes being denoted respectively  $V_\omega$  and  $V_\Omega$ , so:

$$v(0, t) = V_\omega \sin \omega t + V_\Omega \sin \Omega t \quad . \quad (13)$$

A solution of equation (5) subject to the boundary condition (13) can be written as the sum of two terms:

$$v(x, t) = v_\omega(x, t) + v_\Omega(x, t) , \quad (14)$$

leading to (8):

$$v(x, t) = V_\omega \sin \omega \left( t \mp \frac{x}{c_0 \left( 1 \pm \frac{\gamma+1}{2} \frac{v}{c_0} \right)} \right) + V_\Omega \sin \Omega \left( t \mp \frac{x}{c_0 \left( 1 \pm \frac{\gamma+1}{2} \frac{v}{c_0} \right)} \right) \quad (15)$$

We lay

$$u = t \mp \frac{x}{c_0 \left( 1 \pm \frac{\gamma+1}{2} \frac{v}{c_0} \right)} \quad (16)$$

This result follows from the two partial derivatives given below, namely the time derivative:

$$\frac{\partial v}{\partial t} = \left( 1 + \frac{x \frac{\gamma+1}{2} \frac{\partial v}{\partial t}}{c_0^2 \left( 1 \pm \frac{\gamma+1}{2} \frac{v}{c_0} \right)^2} \right) (V_\omega \omega \cos \omega u + V_\Omega \Omega \cos \Omega u) \quad (17)$$

or

$$\frac{\partial v}{\partial t} \left( 1 - \frac{x \frac{\gamma+1}{2} (V_\omega \omega \cos \omega u + V_\Omega \Omega \cos \Omega u)}{c_0^2 \left( 1 \pm \frac{\gamma+1}{2} \frac{v}{c_0} \right)^2} \right) = (V_\omega \omega \cos \omega u + V_\Omega \Omega \cos \Omega u) \quad (18)$$

and the spatial derivative:

$$\frac{\partial v}{\partial x} = \left( \mp \frac{1}{c_0 \left( 1 \pm \frac{\gamma+1}{2} \frac{v}{c_0} \right)} + \frac{x \frac{\gamma+1}{2} \frac{\partial v}{\partial x}}{c_0^2 \left( 1 \pm \frac{\gamma+1}{2} \frac{v}{c_0} \right)^2} \right) (V_\omega \omega \cos \omega u + V_\Omega \Omega \cos \Omega u) \quad (19)$$

or

$$\frac{\partial v}{\partial x} \left( 1 - \frac{x \frac{\gamma+1}{2} (V_\omega \omega \cos \omega u + V_\Omega \Omega \cos \Omega u)}{c_0^2 \left( 1 \pm \frac{\gamma+1}{2} \frac{v}{c_0} \right)^2} \right) = \mp \frac{(V_\omega \omega \cos \omega u + V_\Omega \Omega \cos \Omega u)}{c_0 \left( 1 \pm \frac{\gamma+1}{2} \frac{v}{c_0} \right)} \quad (20)$$

Invoking equation (18) and (20) shows straightforwardly that the solution (15) satisfies equation (5) and (13). This result leads to the approximate solution for both outgoing expected waves, namely

$$v_\Omega(x, t) = V_\Omega \sin \Omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\Omega + v_\omega}{c_0} \right) \right\} , \quad (21)$$

$$v_\omega(x, t) = V_\omega \sin \omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\Omega + v_\omega}{c_0} \right) \right\} . \quad (22)$$

Then, the general solution can take successively the following forms, discarding the second order term of the small quantity  $\frac{\gamma+1}{2} \frac{v}{c_0}$  :

$$v(x, t) = v_\Omega(x, t) + v_\omega(x, t) \quad (23)$$

$$\begin{aligned} &\simeq V_\Omega \sin \Omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\Omega}{c_0} \right) + \frac{x}{c_0} \frac{\gamma+1}{2} \frac{v_\omega}{c_0} \right\} \\ &\quad + V_\omega \sin \omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\omega}{c_0} \right) + \frac{x}{c_0} \frac{\gamma+1}{2} \frac{v_\Omega}{c_0} \right\} \\ &= V_\Omega \left[ 1. \sin \Omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\Omega}{c_0} \right) \right\} \right] \\ &\quad + V_\Omega \left[ \frac{\Omega x}{c_0} \frac{\gamma+1}{2} \frac{v_\omega}{c_0} \cos \Omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\Omega}{c_0} \right) \right\} \right] \\ &\quad + V_\omega \left[ 1. \sin \omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\omega}{c_0} \right) \right\} \right] \\ &\quad + V_\omega \left[ \frac{\omega x}{c_0} \frac{\gamma+1}{2} \frac{v_\Omega}{c_0} \cos \omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\Omega}{c_0} \right) \right\} \right] \\ &\simeq V_\Omega \sin \Omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\Omega}{c_0} \right) \right\} \\ &\quad + V_\Omega v_\omega \frac{\gamma+1}{2c_0^2} \Omega x \cos \Omega \left( t - \frac{x}{c_0} \right) \\ &\quad + V_\omega \sin \omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\omega}{c_0} \right) \right\} \\ &\quad + V_\omega v_\Omega \frac{\gamma+1}{2c_0^2} \omega x \cos \omega \left( t - \frac{x}{c_0} \right) . \end{aligned} \quad (24)$$

The last form of the solution emphasizes that the particle velocity wave can be interpreted as the superposition of four components. Two of them, the first one and the third one, which represent harmonic waves  $v_\Omega$  and  $v_\omega$  (angular frequency respectively  $\Omega$  and  $\omega$ ) propagating independently, have exactly the same behaviour as the Fubini's solutions abovementioned (11), namely:

$$\begin{aligned} S_\Omega &= V_\Omega \sin \Omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\Omega}{c_0} \right) \right\} \\ &= V_\Omega \sum_{n=1}^{\infty} \frac{2}{n\sigma_\Omega} J_n(n\sigma_\Omega) \sin \left\{ n\Omega \left( t - \frac{x}{c_0} \right) \right\} , \end{aligned} \quad (25)$$

$$S_\omega = V_\omega \sin \omega \left\{ t - \frac{x}{c_0} \left( 1 - \frac{\gamma+1}{2} \frac{v_\omega}{c_0} \right) \right\}$$

$$= V_\omega \sum_{n=1}^{\infty} \frac{2}{n\sigma_\omega} J_n(n\sigma_\omega) \sin \left\{ n\omega \left( t - \frac{x}{c_0} \right) \right\} . \quad (26)$$

The other components of the solution (24), the second one and the fourth one, result both from the coupling of the two primary coherent waves created by the acoustic source and propagating nonlinearly due to nonlinear effects. Invoking respectively expression (26) for  $S_\omega$  and expression (25) for  $S_\Omega$ , these second and fourth components take the following forms:

$$\begin{aligned} D_{\Omega, n\omega} &= V_\Omega v_\omega \frac{\gamma+1}{2c_0^2} \Omega x \cos \Omega \left( t - \frac{x}{c_0} \right) \\ &= V_\Omega V_\omega \frac{\gamma+1}{2c_0^2} \Omega x \cos \Omega \left( t - \frac{x}{c_0} \right) \sum_{n=1}^{\infty} \frac{2}{n\sigma_\omega} J_n(n\sigma_\omega) \sin n\omega \left( t - \frac{x}{c_0} \right) \\ &= V_\Omega V_\omega \frac{\gamma+1}{2c_0^2} \Omega x \sum_{n=1}^{\infty} \frac{2}{n\sigma_\omega} J_n(n\sigma_\omega) \cos \Omega \left( t - \frac{x}{c_0} \right) \sin n\omega \left( t - \frac{x}{c_0} \right) \\ &= V_\Omega V_\omega \frac{\gamma+1}{2c_0^2} \Omega x \sum_{n=1}^{\infty} \frac{1}{n\sigma_\omega} J_n(n\sigma_\omega) \left( \sin(\Omega + n\omega) \left( t - \frac{x}{c_0} \right) + \sin(\Omega - n\omega) \left( t - \frac{x}{c_0} \right) \right) \end{aligned} \quad (27)$$

$$\begin{aligned} D_{\omega, n\Omega} &= V_\omega v_\Omega \frac{\gamma+1}{2c_0^2} \omega x \cos \omega \left( t - \frac{x}{c_0} \right) \\ &= V_\Omega V_\omega \frac{\gamma+1}{2c_0^2} \omega x \cos \omega \left( t - \frac{x}{c_0} \right) \sum_{n=1}^{\infty} \frac{2}{n\sigma_\Omega} J_n(n\sigma_\Omega) \sin n\Omega \left( t - \frac{x}{c_0} \right) \\ &= V_\Omega V_\omega \frac{\gamma+1}{2c_0^2} \omega x \sum_{n=1}^{\infty} \frac{2}{n\sigma_\Omega} J_n(n\sigma_\Omega) \cos \omega \left( t - \frac{x}{c_0} \right) \sin n\Omega \left( t - \frac{x}{c_0} \right) \\ &= V_\Omega V_\omega \frac{\gamma+1}{2c_0^2} \omega x \sum_{n=1}^{\infty} \frac{1}{n\sigma_\Omega} J_n(n\sigma_\Omega) \left( \sin(\omega + n\Omega) \left( t - \frac{x}{c_0} \right) + \sin(\omega - n\Omega) \left( t - \frac{x}{c_0} \right) \right) \end{aligned} \quad (28)$$

These results highlight the generation of signals at frequencies  $(\omega \pm n\Omega)$  and  $(\Omega \pm n\omega)$ ,  $n = 1, 2, 3, \dots$ , showing the intermodulation effects which are discussed in section 3.

It is noteworthy that results presented on the particle velocity give straightforwardly similar results on

the acoustic pressure, thanks to the well-known relationship between both i.e. Euler equation which can be reduced to the linear expression when used locally for each frequency of interest ( $\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x}$ ), leading to the same conclusions as those obtained for the particle velocity.